Hot Southern Mathematics

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What is an REU?

- REU stands for Research Experience for Undergraduates.
- Colleges and Universities from all over the United States sponsor students to come their institution and participate in a research experience similar to graduate school.
- Try to encourage students from smaller schools to go onto graduate school.
- Gear towards students from small colleges where it is difficult to get a research position.

Why do an REU?

- Gives you a good idea of what graduate school is like and what professors expect.
- Employers look at it as a good group experience.
- Get to meet people from all over the US interested in the same topic.
- It is an excellent summer job (~$3000 for 8 weeks plus airfare, food, housing).

Selecting and Gathering Information on REUs

- Google REU and the subject .
- Look at a college's departmental website.
- Look at www.augsburg/math/summer-research.html
- A lot of research outside of REUs, which have the same goal, but tend to be harder to find.

What You Need to Get Accepted

- Very different for each school, usually specify what they want.
- Good idea to have a proof based class (Abstract Algebra or Real Analysis), most of material was presented in the same manner.
- Good to have extra skills. I know how to program which turned out to be a huge asset in our project.
- Positions at REUs are very competitive, you need to apply to many places and start early (Start in December or January).
- Apply your Sophomore or Junior year.

My Experience

- Two months of very intense mathematics.
- Different atmosphere than just going to school because you have a lot of freedom as to what you do.
- A lot of the professors time was devoted to helping us.
- Got to compare my education with other schools.
- Worked with some interesting people. Got a feel for what the “Reclusive, antisocial Math Professor” is like.
- I got to travel to Starkville Mississippi, Memphis, Vicksville and New Orleans.

Inverse Scattering of the One-Dimensional Schrödinger Equation with Rational Scattering Coefficients

What is an Inverse Problem?

An Inverse Problem is a problem where we know the answer but we need to find the input.

Example:

$$\text{If } f(x) = y$$

$$\text{Then } f^{-1}(y) = x$$

A similar idea is used in Inverse Scattering.

What is Inverse Scattering?

Inverse scattering problem: the problem of determining the characteristics of an object (its shape, internal constitution, etc.) from measurement data of radiation or particles scattered from the object.(1)

We were examining the plasma wave equation:

$$U_{xx} - U_{tt} = V(x)\,U$$

1: www.thefreedictionary.com

Time Domain to Frequency Domain

We used a Fourier transform on the plasma wave equation to change it from the time domain to the frequency domain.

Time Domain: shows how a signal changes over time.

Frequency domain: shows how much of a signal lies within each frequency band over a range of frequencies.

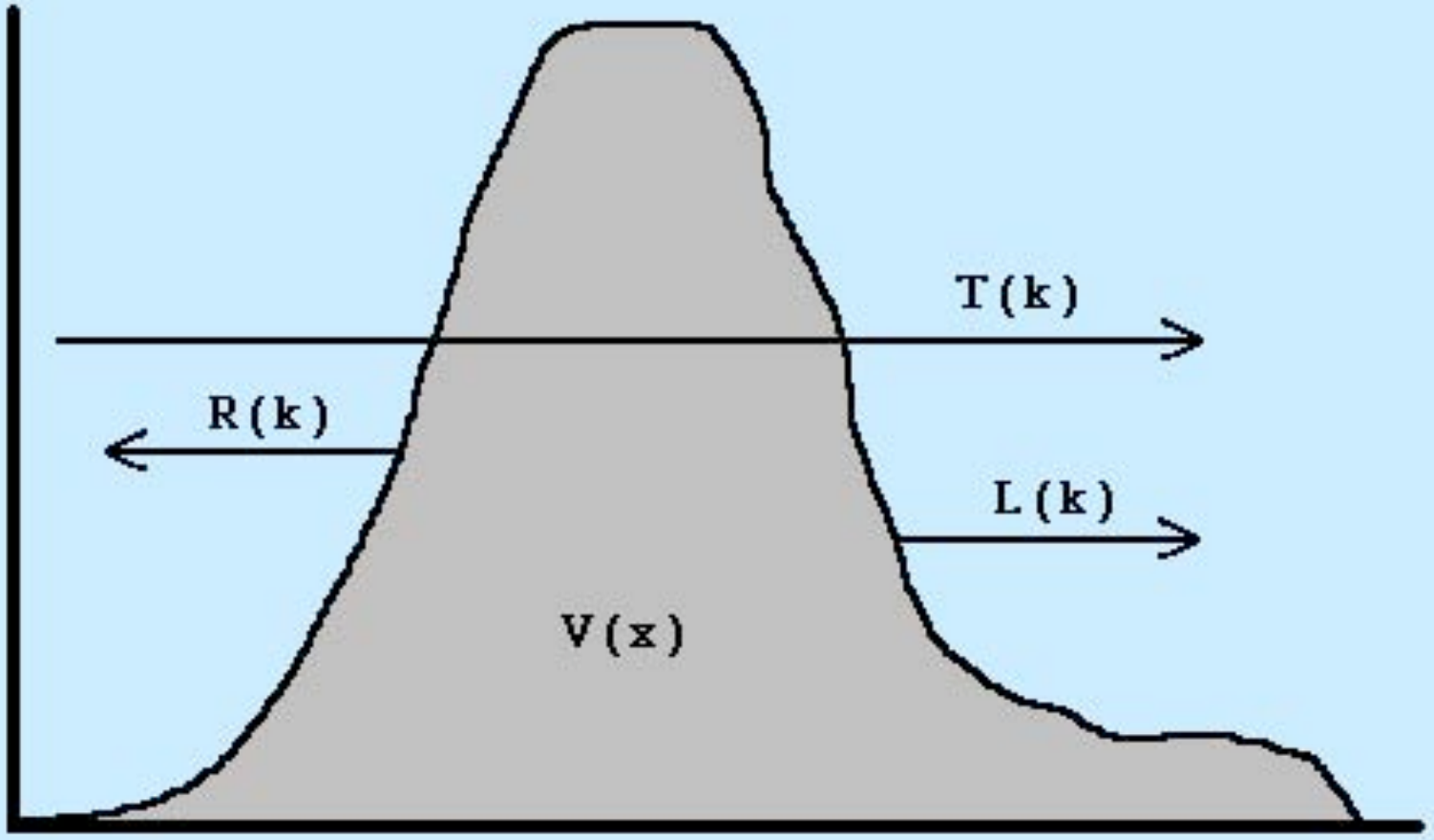
$$U_{xx} - U_{tt} = V(x)\,U \rightarrow \Psi_{xx} - k^2\,\Psi = V(x)\,\Psi$$

Reasons for Changing Domains

- By changing the equation into the frequency domain we could look at the equation as an ordinary differential equation.
- We were also able to look at specific derived quantities and from those quantities use Inverse Scattering.
- Our project was given the **Right Reflection Coefficient** [R(k)] and part of the **Transmission Coefficient** [T(k)], find the potential and all other derived quantities.

In other words, given a reflecting wave we can figure out what the surface it bounced off looks like.

Recovering the Potential



R(k): Right Reflection Coefficient
T(k): Transmission Coefficient
L(k): Left Reflection Coefficient
V(x): Potential

How We Solved the Problem

The process by which we solved the problem was very complex.

Math Used:

Calculus, Linear Algebra, Partial Differential Equations, Ordinary Differential Equations, Real Analysis, Complex Analysis.

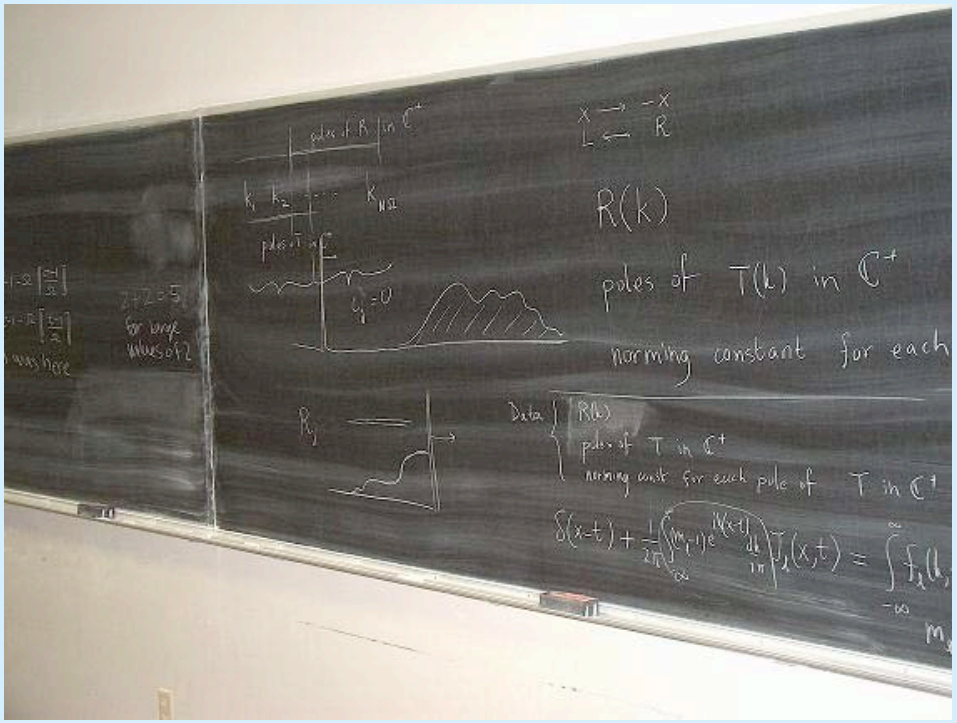
Programming:

Created a program to take in information about the R(k) and return information about the potential.

Used Mathematica (Difficult to use for procedure programming, but it had a lot of nice build in functions that we could not do without.)

Research Difficulties

- 2 week crash courses in high level mathematics.
- Code developed during the previous year could not be expanded upon.
- Mathematica is difficult to program in.
- A lot of the research was a tedious linear algebra problem.



Reasons for the Research

- Generalize research done at Mississippi State University last year.
- Develop more theory and methods for wave research.

Practical Uses

- Process geological data
- Study acoustics (Speech Recognition)
- Radar
- Remote Sensing (military use)

Future Research

- The program needs to be finished and more features added.
- Characterize the one-dimensional plasma wave equation with even less known about it.
- Inverse Scattering is a hot topic in applied mathematics and there are plenty of areas open for research.

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