“I’d rather be approximately right than precisely wrong”: Moving beyond mathematicians natural obsession with the exact in college algebra.

Dr. Suzanne Dorée
Augsburg College, Minneapolis

MAA Contributed Paper Session:
College Algebra: Focusing on Conceptual Understanding, Real-World Data, and Mathematical Modeling
Joint Mathematics Meetings, Washington, DC

January 8, 2009

Introduction

Today's talk

- Augsburg College & the *Applied Algebra* course
- Approximations: an example of what we do differently

Applied Algebra at Augsburg College

Augsburg College

- 4-year private (Lutheran), urban (Minneapolis)
- Liberal arts + Business + Education
- $\approx$ 3000 undergraduates: traditional + adult
- $\approx$ 40% need *Applied Algebra*
- Diverse – soc/eco (1st gen), race/ethnicity, age, disabilities

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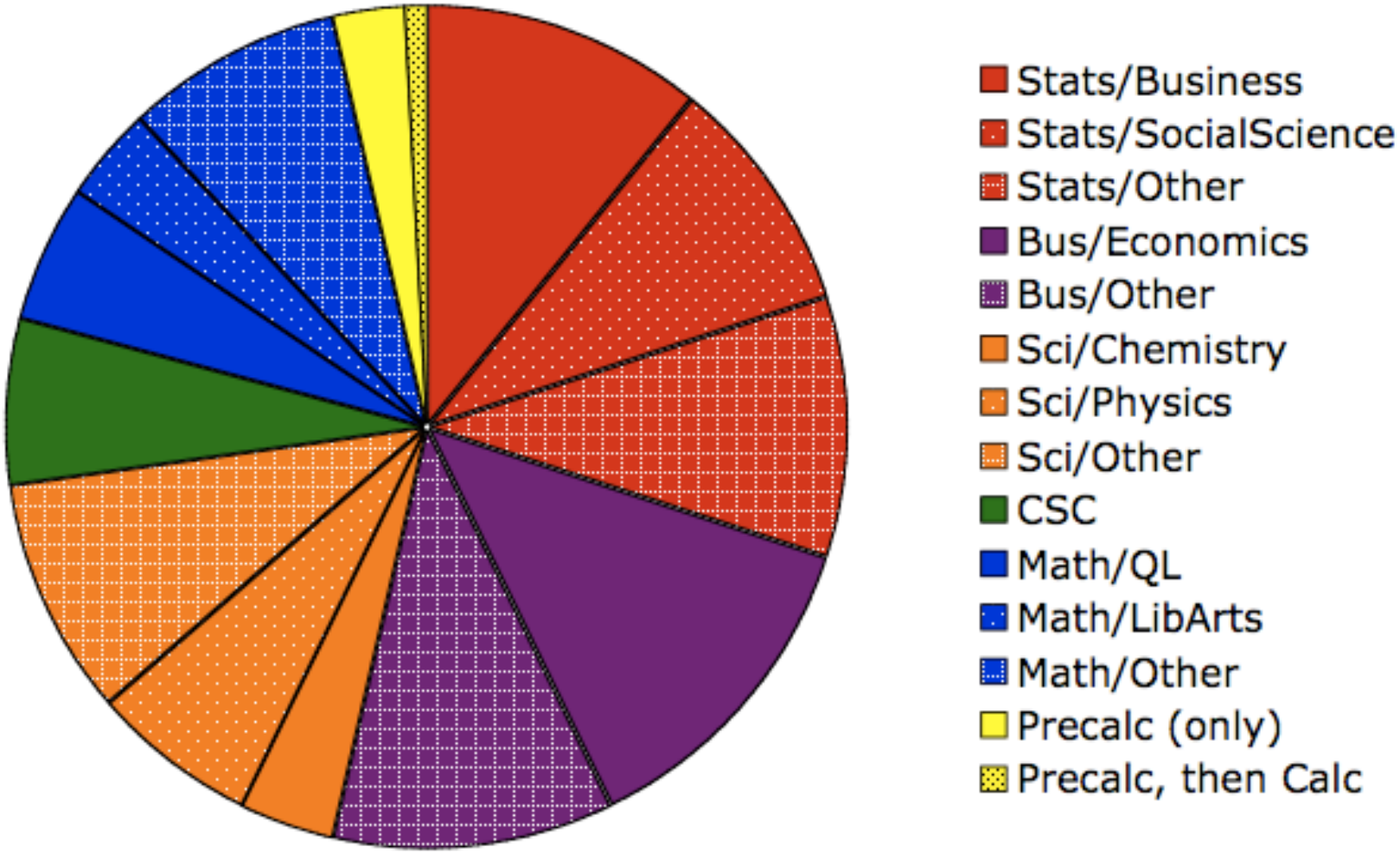
Typical *Applied Algebra* student

- 3-4 years of HS math but did not go well.
- Only takes class because required.
- Does not like math.
- Many challenges – years away, learning disability, physical disability, math anxiety, poor study skills, low responsibility, at risk behaviors, missing support systems.

Learning objectives based on where our students go next



Learning objectives and content

- Based on consultations with client disciplines – sciences, business, stats, math
- Agree (mainly) with recommendations of professional orgs
  - MAA CUPM Curriculum Guide 2004
  - MAA/CRAFTY *Curriculum Foundations Project: Voices of the Partner Disciplines*
  - MAA *A Fresh Start for Collegiate Mathematics*
  - *Mathematics and Democracy: The Case for Quantitative Literacy*, Steen
  - AMATYC *Crossroads* & *Beyond Crossroads*
  - NCTM Standards for K-12

Applied Algebra at Augsburg College

Learning objectives

- Perform key skills
  - Calculator, order of operations, decimals, %
  - Units, reasonableness of answers
  - Interpret table, graph, equation
  - Create table, graph, equation
- Understand key concepts
  - Variable, function
  - Equation, solution
  - Linearity, exponential change, rate of change
- Recognize mathematics in applied contexts
- Be confident in ability to do the mathematics

Course content

- Unit 1 Big ideas of algebra
  - Variables, tables, graphs
  - Units, scientific notation, approximation
  - Intro to equations, numerical/graphical/symbolic solutions
  - Rate of change
- Unit 2 Linear models
- Unit 3 Exponential models
- Mini-unit Other models

Applied Algebra at Augsburg College

Taught 100% in applied contexts from

- Subsequent courses – esp. business, science, social sci
- Everyday life – health, personal finance, . . .
- Math for Citizenship – in the news

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  - Learn some important mathematics.

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- Faculty
  - Like the class – useful, relevant, interesting. Fun to teach.
  - Not too much work.
  - Very rewarding.

Approximations: an example of what we do differently

A few of our learning objectives

- Evaluate reasonableness of answers
- Understand concept of solution
- Recognize mathematics in applied contexts

Approximations: an example of what we do differently

Exercises from the first day

1. Social Security benefits increase 2.7% per year.
   In 2002 my grandmother's monthly benefit was $326.17
   What is her monthly benefit now?

2. The temperature was 40° yesterday at noon,
   but it dropped 3°/hour after noon.
   When did the temp fall below freezing (32°)?

3. A twenty pound bag of dog food costs $12.95,
   but a five pound bag costs $3.79.
   How much expect ten pound bag costs?

4. My car was worth $22,500 when I bought it.
   Now it's ten years old and worth only $7,500.
   When fall below $500 in value?

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Exercises from the first day ..... **Su's answers**

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Approximations: an example of what we do differently

With common sense and some algebra
you can understand the world better than you can
with common sense alone.

Prof. Ethan Bolker, U. Mass Boston in *Using Algebra*

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Two quotes

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